Solubility Product

1. a) $CaF_2(s) \rightleftharpoons Ca^{2+}(aq) + 2F^-(aq)$

$$K_{sp} = [Ca^{2+}][F^-]^2$$

b) $PbSO_4(s) \rightleftharpoons Pb^{2+}(aq) + SO_4^{2-}(aq)$

$$K_{sp} = [Pb^{2+}][SO_4^{2-}]$$

c) $Ag_2CO_3(s) \rightleftharpoons 2Ag^+(aq) + CO_3^{2-}(aq)$

$$K_{sp} = [Ag^+]^2[CO_3^{2-}]$$

2. $BaSO_4(s) \rightleftharpoons Ba^{2+}(aq) + SO_4^{2-}(aq)$

I	0	0
C	+x	+x
E	x	x

solubility $BaSO_4 = [Ba^{2+}] = [SO_4^{2-}]$

$$\text{solubility} = 0.00245\ \frac{g\ BaSO_4}{L} \times \frac{1\ mol\ BaSO_4}{233.40} = 1.05 \times 10^{-5}\ \frac{mol}{L}$$

$\therefore [Ba^{2+}]_{eq'm} = [SO_4^{2-}]_{eq'm} = 1.05 \times 10^{-5}$ mol/L

$$K_{sp} = [Ba^{2+}][SO_4^{2-}]$$
$$= (1.05 \times 10^{-5})(1.05 \times 10^{-5})$$
$$K_{sp} = 1.10 \times 10^{-10}$$

3. $\frac{0.649\ g\ CaCrO_4}{156\ mL} = \frac{x\ g}{1000\ mL}$

$x = \frac{4.16\ g\ CaCrO_4}{1000\ mL}$

$\text{solubility } CaCrO_4 = 4.16\ g\ \frac{CaCrO_4}{L} \times \frac{1\ mol\ CaCrO_4}{156.08\ g\ CaCrO_4}$

$= 0.0267\ mol\ \frac{CaCrO_4}{L}$

$CaCrO_{4(s)} \rightleftharpoons Ca^{2+}_{(aq)} + CrO_4^{2-}{}_{(aq)}$

I	0	0
C	+x	+x
E	x	x

$\text{solubility } CaCrO_4 = [Ca^{2+}]_{eq'm} = [CrO_4^{2-}]_{eq'm} = 0.0267\ \frac{mol}{L}$

$K_{sp} = [Ca^{2+}][CrO_4^{2-}]$

$= (0.0267)(0.0267)$

$= 7.1 \times 10^{-4}$

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4. LiF has higher K_{sp} ∴ ↑ solub.

$LiF_{(s)} \rightleftharpoons Li^+_{(aq)} + F^-_{(aq)}$

I	0	0
C	+x	+x
E	x	x

$K_{sp} = [Li^+][F^-]$

$1.7 \times 10^{-3} = (x)(x)$

$x = 0.041$

$\text{solubility } LiF = [F^-]_{eq'm}$

$= [Li^+]_{eq'm}$

$= 0.041\ mol/L$

$BaF_{2(s)} \rightleftharpoons Ba^{2+}_{(aq)} + 2F^-_{(aq)}$

I	0	0
C	+x	+2x
E	x	2x

$K_{sp} = [Ba^{2+}][F^-]^2$

$1.7 \times 10^{-6} = (x)(2x)^2$

$4x^3 = 1.7 \times 10^{-6}$

$x = 7.5 \times 10^{-3}$

$\text{solubility } BaF_2 = [Ba^{2+}] = \frac{1}{2}[F^-]$

$= 7.5 \times 10^{-3}\ mol/L$

5.

$$AgCN_{(s)} \rightleftharpoons Ag^+_{(aq)} + CN^-_{(aq)}$$

I	0	0
C	+x	+x
E	x	x

solubility $AgCN = [Ag^+]$
$= [CN^-]$
$= 1.48 \times 10^{-8} \frac{mol}{L}$

$K_{sp} = [Ag^+][CN^-]$

$2.2 \times 10^{-16} = (x)(x)$

$x = 1.48 \times 10^{-8}$

sol' $AgCN = 1.48 \times 10^{-8} \frac{mol}{L} \times \frac{133.89\ g}{1\ mol}$

$= \frac{1.98 \times 10^{-6}\ g\ AgCN}{1\ L}$

$= \frac{1.98 \times 10^{-7}\ g\ AgCN}{100\ mL}$

$$Zn(CN)_{2\,(s)} \rightleftharpoons Zn^{2+}_{(aq)} + 2CN^-_{(aq)}$$

I	0	0
C	+x	+2x
E	x	2x

$K_{sp} = [Zn^{2+}][CN^-]^2$

$3.0 \times 10^{-16} = (x)(2x)^2$

$4x^3 = 3.0 \times 10^{-16}$

$x = 4.2 \times 10^{-6}$

solubility $Zn(CN)_2 = [Zn^{2+}] = \frac{1}{2}[CN^-]$

$= 4.2 \times 10^{-6} \frac{mol}{L} \times \frac{117.43\ g}{1\ mol\ Zn(CN)_2}$

$= 4.9 \times 10^{-4} \frac{g\ Zn(CN)_2}{L}$

$= \frac{4.9 \times 10^{-5}\ g\ Zn(CN)_2}{100\ mL}$

6 a) $CuCl_{(s)} \rightleftharpoons Cu^+_{(aq)} + Cl^-_{(aq)}$

I	0	0
C	$+x$	$+x$
E	x	x

$K_{sp} = [Cu^+][Cl^-]$

$1.9 \times 10^{-7} = (x)(x)$

$x = 4.4 \times 10^{-4}$

solubility $= [Cu^+] = [Cl^-]$

$= 4.4 \times 10^{-4}$ mol/L

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b) $CuCl_{(s)} \rightleftharpoons Cu^+_{(s)} + Cl^-_{(aq)}$

I	0	0.02
C	$+x$	$+x$
E	x	$0.02 + x$

$[Cl^-]_I = [HCl] \times$ mol ratio

$= 0.02 \frac{mol\ HCl}{1L} \times \frac{1\ mol\ Cl^-}{1\ mol\ HCl}$

$= 0.02 \frac{mol\ Cl^-}{L}$

check

$\frac{[\ ]_I}{K_{sp}}$

$= \frac{0.2}{1.9 \times 10^{-7}}$

$>> 500$

$\therefore 0.02 + x$

≈ 0.02

$K_{sp} = [Cu^+][Cl^-]$

$1.9 \times 10^{-7} = (x)(0.2)$

$x = 9.5 \times 10^{-6}$

proof

$\frac{x}{[\ ]_I} \times 100\%$

$\frac{9.5 \times 10^{-6}}{0.02} \times 100\%$

$= 0.0475\%$

$\therefore$ ass. valid

solubility $CuCl = [Cu^+]$

$= 9.5 \times 10^{-6} \frac{mol}{L}$

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c) $CuCl_{(s)} \rightleftharpoons Cu^+_{(aq)} + Cl^-_{(aq)}$

I	0	0.3
C	$+x$	$+x$
E	x	$x + 0.3$

$K_{sp} = [Cu^+][Cl^-]$

$1.9 \times 10^{-17} = x\ (0.3)$

$x = 6.3 \times 10^{-17}$

solubility $CuCl = [Cu]$

$= 6.3 \times 10^{-17} \frac{mol}{L}$

check

$\frac{[\ ]_I}{K_{sp}} = \frac{0.3}{1.9 \times 10^{-17}}$

$>> 500$

$\therefore 0.3 + x \approx 0.3$

proof

$\frac{x}{[\ ]_I} \times 100\%$

$\frac{6.3 \times 10^{-17}}{0.3} \times 100\% < 5\%$

7. a) $Mg(OH)_2\ (s) \rightleftharpoons Mg^{+}\ (aq) + 2OH^{-}\ (aq)$

I	0	0
C	+x	+2x
E	x	2x

$K_{sp} = [Mg^{2+}][OH^-]^2$

$7.1 \times 10^{-12} = (x)(2x)^2$

$4x^3 = 7.1 \times 10^{-12}$

$x = 1.2 \times 10^{-4}$

solubility $Mg(OH)_2 = [Mg^{2+}]$

$= \frac{1}{2}[OH^-]$

$= 1.2 \times 10^{-4}\ \frac{mol}{L}$

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b) $Mg(OH)_2\ (s) \rightleftharpoons Mg^{2+}\ (aq) + 2OH^-\ (aq)$

I	0.25	0
C	+x	+2x
E	0.25+x	2x

check $\frac{Q}{K_{sp}}$

$\frac{0.25}{1.7 \times 10^{-12}} >> 500$

$K_{sp} = [Mg^{2+}][OH^-]^2$

$1.7 \times 10^{-12} = (0.25)(2x)^2$

$4x^2 = 6.8 \times 10^{-12}$

$x = 1.3 \times 10^{-6}$

proof

$\frac{x}{[\ ]_I} \times 100\%$

$\frac{1.3 \times 10^{-6}}{0.25} \times 100\% < 5\%$

solubility $Mg(OH)_2 = \frac{1}{2}[OH^-]$

$= 1.3 \times 10^{-6}\ \frac{mol}{L}$

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c) $Mg(OH)_2\ (s) \rightleftharpoons Mg^{2+}\ (aq) + 2OH^-\ (aq)$

I	0	0.20
c	+x	+2x
E	x	0.2+2x

Check

$\frac{0.2}{K_{sp}} >> 500$

proof

$\frac{x}{[\ ]_I} \times 100\%$

$\frac{4.25 \times 10^{-4}}{0.2} \times 100\% < 5\%$

$K_{sp} = [Mg^{2+}][OH^-]^2$

$1.7 \times 10^{-12} = x\ (0.2)^2$

$x = 4.25 \times 10^{-11}$

solubility $Mg(OH)_2 = [Mg^{2+}]$

$= 4.25 \times 10^{-11}\ \frac{mol}{L}$

8. $Pb(NO_3)_{2(aq)} + 2NaCl_{(aq)} \rightarrow PbCl_{2(s)} + 2NaNO_{3(aq)}$

$PbCl_{2(s)} \rightleftharpoons Pb^{2+}_{aq} + 2Cl^-_{(aq)}$

$[Pb^{2+}] = [Pb(NO_3)_2] \times \text{mol ratio} \times \frac{V_{Pb(NO_3)_2}}{V_T}$

$= 0.015 \text{ mol } \frac{Pb(NO_3)_2}{L} \times \frac{1 \text{ mol } Pb^{2+}}{1 \text{ mol } Pb(NO_3)_2} \times \frac{35.6 \text{ mL}}{59.28 \text{ mL}}$

$= 0.009 \text{ mol } \frac{Pb^{2+}}{L}$

$[Cl^-] = [NaCl] \times \text{mol ratio} \times \frac{V_{NaCl}}{V_T}$

$= 0.012 \text{ mol } \frac{NaCl}{L} \times \frac{1 \text{ mol } Cl^-}{1 \text{ mol } NaCl} \times \frac{23.68 \text{ mL}}{59.28 \text{ mL}}$

$= 0.00479 \text{ mol } \frac{Cl^-}{L}$

$Q_{sp} = [Pb^{2+}][Cl^-]^2$

$= (0.009)(0.00479)^2$

$= 2.07 \times 10^{-7}$

$Q_{sp} < K_{sp}$ ∴ no ppt

9. $Pb(NO_3)_2\,(aq) + 2KBr\,(aq) \rightarrow PbBr_2\,(aq) + 2\,KNO_3\,(aq)$

(a) $PbBr_2\,(s) \rightleftharpoons Pb^{2+}\,(aq) + 2Br^-\,(aq)$

$[Pb^{2+}] = [Pb(NO_3)_2] \times \text{mol ratio} \times \frac{V}{V_T}$

$= 0.01\ \text{mol}\ Pb(NO_3)_2 \times \frac{1\ \text{mol}\ Pb^{2+}}{1\ \text{mol}\ Pb(NO_3)_2} \times \frac{50.0\ \text{mL}}{100\ \text{mL}}$

$= 0.005\ \text{mol}\ Pb^{2+}$

$[Br^-] = [KBr] \times \text{mol ratio} \times \frac{V}{V_T}$

$= 0.01\ \text{mol}\ KBr \times \frac{1\ \text{mol}\ Br^-}{1\ \text{mol}\ KBr} \times \frac{50.00\ \text{mL}}{100\ \text{mL}}$

$= 0.005\ \text{mol}\ Br^-$

$Q_{sp} = [Pb^{2+}][Br^-]^2$

$= (0.005)(0.005)^2$

$= 1.25 \times 10^{-7}$

$Q_{sp} < K_{sp}$

$\therefore$ no ppt

(b) $Pb(NO_3)_2\,(aq) + CaBr_2\,(aq) \rightarrow PbBr_2\,(s) + Ca(NO_3)_2\,(aq)$

$PbBr_2\,(s) \rightleftharpoons Pb^{2+}\,(aq) + 2Br^-\,(aq)$

$[Pb^{2+}] = [Pb(NO_3)_2] \times \text{mol ratio} \times \frac{V}{V_T}$

$= 0.0100\ \text{mol}\ Pb(NO_3)_2 \times \frac{1\ \text{mol}\ Pb^{2+}}{1\ \text{mol}\ Pb(NO_3)_2} \times \frac{50.0\ \text{mL}}{125.0\ \text{mL}}$

$= 4.00 \times 10^{-3}\ \text{mol}\ Pb^{2+}$

$[Br^-] = [CaBr_2] \times \text{mol ratio} \times \frac{V}{V_T}$

$= 0.0526\ \text{mol}\ CaBr_2 \times \frac{2\ \text{mol}\ Br^-}{1\ \text{mol}\ CaBr_2} \times \frac{75.0\ \text{mL}}{125\ \text{mL}}$

$= 6.312 \times 10^{-2}\ \text{mol/L}$

$Q_{sp} = [Pb^{2+}][Br^-]^2$

$= (4 \times 10^{-3})(6.312 \times 10^{-2})^2$

$= 1.6 \times 10^{-5}$

$Q_{sp} > K_{sp}$

$\therefore$ ppt